 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2012**

# MT 1816 - REAL ANALYSIS

(12 BATCH STUDENTS ONLY)

Date : 05/11/2012 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

**Answer all the questions. Each question carries 20 marks.**

**I.a) 1)** Prove that refinement over an interval increases the lower sum and decreases the upper sum.

**OR**

**a)2.** Using the notion of Upper sums and Lower sums of a bounded function, when do you say that  on [a,b]. When does the Riemann-Stieltjes integral reduces to Riemann integral? (5)

**b)1)** Suppose f is bounded on [a,b]. f has finitely many points of discontinuity on [a,b] and is continuous at every point at which f is discontinuous the prove that .

**b)2)** Prove: Let  be a monotonically increasing function on [a,b] and let on [a,b]. If f is a bounded real function on [a,b] then prove that on [a,b] on [a,b]. In this case  (5 +10)

**OR**

**c)1)** Prove: on [a,b] 

**c)2)** Suppose c ∈ (a,b) and two of the three integrals ,  and  exist. Then prove that the third also exists and 

**c)3)** If f is monotonic on [a,b] and if  is continuous on [a,b], then prove that  . (4+4+7)

**II. a)1)** State and prove the theorem on Cauchy criterion for uniform convergence.

**OR**

**a)2)** Using a suitable example show that the limit of the integral need not be equal to the integral of the limit. (5)

**b)1)** If X is a metric space and denote the set of all complex valued, continuous, bounded functions with domain X. Choosing a suitable distance function between the elements of  prove that is a metric space and also a complete metric space.

**b)2)** Suppose that is a sequence of differentiable functions on [a,b]. Suppose that converges at some point . If converges uniformly on [a,b] then prove that converges uniformly to some function f and  (5+10)

**OR**

**c)1)** Let . Verify whether .

**c)2)** Suppose  converges uniformly to a function f on E, where E is a set as a metric space. Let x be a limit point of E and suppose that . Then prove that {An}conveges and  (5+10)

**III. a)1)** State and prove Parseval’s formula.

**OR**

**a)2)** State and prove Dini’s theorem. (5)

**b)1)** State and prove Jordan’s theorem.

**b)2)**State and prove Riemann- Lebesgue Lemma. (6+9)

**OR**

**c)1)** State and prove Riesz – Fischer’s theorem.

**c)2)** State and prove Riemann Localization theorem. (6+9)

**IV. a)1)** Suppose E is an open set in Rn and **f** maps E into Rm and x is an element in E such that when , . Then prove that A is unique.

**OR**

**a)2)** Let be the set of all invertible linear operators on Rn. If then prove that . (5)

**b)** State and prove Inverse function theorem. (15)

**OR**

**c)1)** If  and c is a scalarthen prove that  . And with the distance between A and B defined as , prove that is a metric space.

**c)2)** Define Contraction principle and prove the following theorem: Let X is a complete metric space and if  is a contraction of X into X then there exists only one xX such that (x)=x. (5+10)

**Va)1)**. Derive the rectilinear co ordinates.

**OR**

**a)2)** Derive the sum of powers of . (5)

**b)1)** Explain how the product and quotient rule are derived for functions f(x) and g(x)?

**OR**

**b)2)** Derive the expression for D’ Alembert’s wave equation for a vibrating string.

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